MATH 2850: 5.6 - REDUCTION OF ORDER

Consider the ODE: $y'' + p(x)y' + q(x)y = f(x)$.

IDEA: Given a nontrivial solution y_1 to the associated homogeneous DE, assume a solution of the form $y = u\,y_1$.

Substituting $y = u\,y_1$ into the DE gives is a **first order linear** DE for u'.

PROOF:

REDUCTION OF ORDER:

- First, 'find' a nontrivial solution to the DE, y_1.
- Second, substitute $y = u\,y_1$ into the DE.
- Third, solve for u' and hence u.
- Find $y = u\,y_1$ to get a second non-trivial solution.

EXAMPLE: Solve $x^4y'' + x^3y' - x^2y = 4$ given $y_1 = x$ is part of the complementary solution.

Ans: $y = c_1x + c_2x^{-1} + \dfrac{4}{3}x^{-2}$

EXAMPLE: Solve $x^2y'' + 2x(x-1)y' + (x^2-2x+2)y = x^3e^{2x}$, given $y_1 = xe^{-x}$ is part of the complementary solution.

Ans: $y = c_1x\,e^{-x} + c_2x^2\,e^{-x} + \dfrac{1}{9}x\,e^{2x}$

EXAMPLE: Solve $x^2y'' - 2xy' + (x^2 + 2)y = 0$, given $y_1 = x\sin(x)$ is part of the complementary solution.

Ans: $y = c_1x\ \sin(x) + c_2x\ \cos(x)$

HOMEWORK: Pg. 253: 1 - 33 every other odd.